Auctions 2: Models and Practice.

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Revenue equivalence theorem: Key assumptions

- Two pairs (mechanism, equilibrium) with the same allocation rule
- Independence of valuations (information)
- Risk neutrality
- No budget constraints
- "No collusion" (correct equilibrium); "no resale" (correct game)

Budget constraints

- Every bidder obtains value (signal) $X_i \in [0,1]$ and absolute budget $W_i \in [0,1]$.
- (X_i, W_i) are iid across bidders. (X_i and W_i need not be independent.)

Proposition: With budget-constrained bidders the expected revenue in a first-price auction is greater than in a second-price auction. (provided symmetric equilibrium exists.)
Intuition: The bids in second-price auction are higher on average and so are more often constrained.
(Not enough: players will reduce bids in the first-price auction).

Proof: In the second-price auction:

$$\beta^{\text{II}}(x, w) = \min\{x, w\}.$$

Define (effective type) $x^{\text{II}} \sim (x, w)$ as the type that is effectively unconstrained and submits the same bid as (x, w). Can be found as a solution to

$$\beta^{\text{II}}(x, w) = \beta^{\text{II}}(x^{\text{II}}, 1) = x^{\text{II}}.$$

Let $Y_2^{\text{II}(N)}$ be the second highest of the equivalent values, x_i^{II}, among N bidders. Its distribution is

$$G^{\text{II}}(z) = \left(F^{\text{II}}(z)\right)^{N-1},$$

where $F^{\text{II}}(z)$ is the probability that
$\beta^{\text{II}}(x, w) = \beta^{\text{II}}(x^{\text{II}}, 1) = x^{\text{II}} < z = \beta^{\text{II}}(z, 1)$.
We have

$$E[R^{\text{II}}] = E\left[Y_2^{\text{II}(N)}\right].$$

In the first-price auction: Suppose a symmetric increasing equilibrium exists with

$$\beta^{\mathrm{I}}(x, w) = \min\{\beta(x), w\}.$$

Define $x^{\mathrm{I}} \sim (x, w)$ as the solution to

$$\beta^{\mathrm{I}}(x, w) = \beta^{\mathrm{I}}(x^{\mathrm{I}}, 1) = \beta(x^{\mathrm{I}}) < x^{\mathrm{I}}.$$

Let $Y_2^{\mathrm{I}(N)}$ be the second highest of the equivalent values, x_i^{I}, among N bidders. Its distribution is

$$G^{\mathrm{I}}(z) = \left(F^{\mathrm{I}}(z)\right)^{N-1}.$$

We have

$$E[R^{\mathrm{I}}] = E\left[Y_2^{\mathrm{I}(N)}\right].$$

Note that $F^{\mathrm{I}}(z) < F^{\mathrm{II}}(z)$, and thus

$$E[R^{\mathrm{I}}] > E[R^{\mathrm{II}}].$$

All-pay auctions dominate first-price auctions in terms of revenue.

Other settings:

- Single-unit auctions: different allocation rules.
  e.g., with reserve price R or participation decisions.
- Multi-unit auctions with identical items.
  Q and q are quantity of items won.
- Bilateral and multilateral trade.
  Q and q is probability of trade (quantity).
- Monopolistic markets (models of discrimination)
  Q and q are quantities of goods sold or quality.
- Optimal taxation/ contractual schemes...

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- With incomplete information, Myerson-Satterthwaite Theorem says that efficient bilateral trade is IMPOSSIBLE
- Efficient privatization auctions exist! (generalized Vickrey mechanism)

Bilateral Trade: setting

Independent private values setting with risk-neutral seller and buyer, no budget constraints.

- Single indivisible object for sale.
- S — valuation of the seller; V — valuation of the buyer.
- $S \sim F_S[0, \omega]$, $V \sim F_V[0, \omega]$ — independent, and private; distributions are common knowledge

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- Vickrey mechanism: Efficient; prices are externalities on society
- Seller: Without him, Buyer $U_B = 0$, with trade, $U_B = V_B$, thus, $P_S = -V$.
- Buyer: Without him, $U_S = 0$; with trade, $U_S = -S$, thus, $P_B = S$.

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- Total transfer: $P_B + P_S = S - V < 0$ (if $V > S$).

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- The best constrained-efficient mechanism?
- Double auction is the second-best for uniform distributions. Double auction gets closer to (full) efficiency as number of participants grows. Moreover, this happens "fast" and Market (Rational-Expectations) equilibrium in the limit.

Russian Privatization

Theoretical problems (Inefficiency):

- Auction mechanism: everyone wins in proportion to her bid
  better: dynamic auction, a la IPO auctions.
- Budget constraints
  better: delay in time, non-monetary auctions (payments spread-out in time).
- Coase Theorem
  better: (careful) efficient design.