Impact of Forward Market on Producers Market Power

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Actuality

Market power of large producers and related reduction of social welfare cause an important theoretical and practical problem. It is of special interest in context of electricity markets development. Splitting of electricity market into small companies is a bad way to deal with the problem because of scale effect and reliability requirements. A more adequate way to reduce market power is introduction of forward market.

Literature overview

Allaz и Vila ***(1993)***: in oligopolistic industry introduction of forward market induces more competitive outcomes.
Hughes и Kao ***(1997)***: this result is achieved under assumption of firms' forward positions being perfectly observed, otherwise Cournot outcome arises.
Mahenc и Salanie ***(2004)***: under Bertrand-Edgeworth competition at the spot market, a possibility of forward contracting may increase market power and reduce social welfare; in equilibrium in such a model each producer buys forwards on its own production in order to increase spot price.

Literature overview

James Bushnell ***(2005)*** considers a symmetric oligopoly and a two-stage market with Cournot competition at the spot market and no-arbitrage condition. For a constatnt marginal cost, he shows that introduction of forward market reduces market power in the same way as increasing number of producers from n to n^2.

His model is based on several unfaithful assumptions:

1. forward and spot prices are equal (see Botterud et al.);
2. individuals with higher reserve prices buy at the forward market.

In the present research we eliminate those assumptions. Based on real markets' statistics, we assume that spot price is random and that forward price is equalized to expected spot price due to arbitrageurs' activity.

Timing of interaction

We consider symmetric oligopoly of n producers with fixed marginal costs c. Producers act at the forward market and then at the spot market, organized as Cournot auctions. We assume the following timing of interaction:

1. Producers set their volumes for forward market.

2. Consumers choose if they are going to buy at the forward market. Each consumer is characterized by utility function $U_b = U(\Delta, \lambda_b)$, where $\Delta = p - r_b$, r_b – reserve price of consumer b, p - cut-off price, λ_b – risk-aversion parameter evenly increasing by Δ, $U(0, \lambda_b) = 0 \; \forall \lambda_b \in [\lambda_{\min}, \lambda_{\max}], \lambda_{\min} < 0 < \lambda_{\max}$. For risk-neutral consumers $\lambda_b = 0$, utility function is linear. For risk-averse consumers $\lambda_b > 0$, utility function is concave. Risk-aversion increases with the raise of λ:

$$\ln U(\Delta, \lambda)''_{\Delta\lambda} \leq 0.$$

Timing of interaction

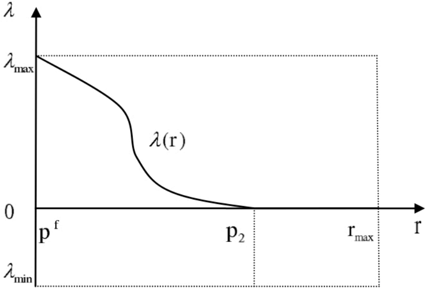
Demand of consumers who would like to participate in the forward market together with supply of producers determine cut-off price for forward market without arbitrageurs. It also determines residual demand at the spot market without arbitrageurs and the correspondent spot prices. Price at the spot market is dependent on an observable random factor $\omega \in \{1, 2\}$: price $p_1 < p^f$ occurs with probability w ("bear market"), price $p_2 > p^f$ – with probability $1 - w$ ("bull market"). In equilibrium $p^f = wp_1 + (1 - w)p_2$.

3. Arbitrageurs set their strategies: to buy forward contracts or to sell forward contracts. We assume perfect competition among arbitrageur, so that their activity equalizes forward price and mathematical expectation of the spot price. The strategy of producer $a, a \in A$ is a pair $q_a = (q^f_{a1}, q^s_a(q^f, \omega))$, where q^f_{a1} is supply volume of a firm at the forward market, $\sum_{a \in A} q^f_{a1} \stackrel{\text{def}}{=} q^f_1$ – total forward supply by producers, q^s_a – supply volume of a firm at the spot market, which depends on total volume q^f sold at the forward market and ω. Each producer aims to maximize its total revenue. $q^f = q^f_1 + q^f_2$, where q^f_2 – total volume sold by arbitrageurs at the forward market ($q^f_2 > 0$)/ bought by arbitrageurs at the forward market ($q^f_2 < 0$).

Optimal behaviour of consumers

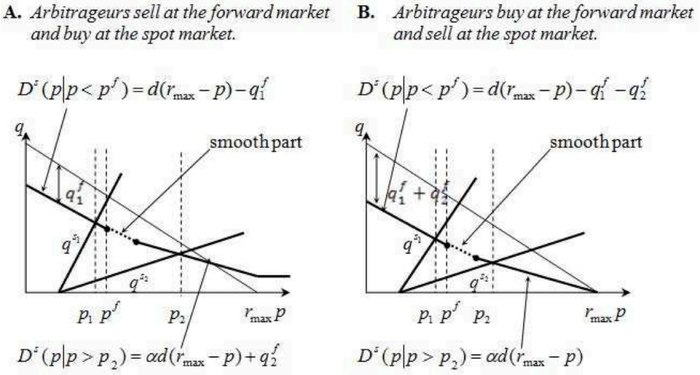
Proposition 1

1. Consumers with reserve prices $p_1 < r_b < p^f$, as well as risk-neutral ($\lambda_b = 0$) or risk-preferring ($\lambda_b < 0$) consumers with $p^f < r_b < p_2$, buy the good only at the spot market under low price p_1.
2. For $p^f < r_b < p_2$ there exists a threshold $\lambda(r)$ such that consumers with $\lambda_b > \lambda(r)$ buy the good at the forward market, and those with $\lambda_b < \lambda(r)$ act as in case *1*; $\lambda(r)$ decreases from $\lambda_{\max}$ to 0 in this interval.
3. For $r_b > p_2$ risk-preferring consumers buy the good at the spot market and risk-averse consumers - at the forward market.



Search of SPE

Let consumers demand at the forward market be linear:
$D(p) = \bar{D} - dp$, $r_{\max} = \frac{\bar{D}}{d}$. Let distribution of consumers be characterized by density function $\rho(r, \lambda)$. Thus, residual demand function at the spot market is:
$D^s(p) = \int\limits_{p}^{r_{\max}} \int\limits_{\lambda_{\min}}^{\lambda(r)} \rho(r, \lambda) d\lambda dr$. Let α be a share of risk-averse consumers with $r_b > p_2$. Thus, residual demand at the spot market is:



SPE prices and production volumes at the spot market

Proposition 2

If there exists SPE in correlated mixed strategies in this model, then SPE prices p_1, p_2 and production volumes q^{s1}, q^{s2} meet the following relations:

$$p_1 = p^* - \frac{q_1^f}{d(n+1)},\ q^{s1} = nd(p_1 - c) = nd(\Delta^* - \frac{q_1^f}{d(n+1)}),$$

$$p_2 = p^* + \frac{q_2^f}{\alpha d(n+1)} > p_1,\ q^{s2} = n\alpha d(\Delta^* + \frac{q_2^f}{\alpha d(n+1)}) < q^{s1},$$

where p^* is a Cournot price for one-stage auction, $\Delta^* = p^* - c = \frac{\bar{D} - dc}{d(n+1)}$.

Thus,
$p^f = wp_1 + (1-w)p_2 = p^* - wq_1^f/d/(n+1) + (1-w)\frac{q_2^f}{\alpha d(n+1)}, w \in [0,1]$.

Optimal production volume at the forward market

Consider situation when risk-averse consumers with $p^f < r_b < p_2$ buy at the forward market. This corresponds to $\lambda_b = \lambda_{\max}$. Proceeding from the F.O.C. for producers' expected profit maximization we obtain:

Proposition 3

In SPE exists in the model, then optimal volumes at the forward market are the following:

$$q_1^f = \frac{\Delta^* d(n+1)(1 + \frac{(1-w)(1-\alpha)n}{K_1} - \frac{2}{n+1}(w + (1-w)(\frac{K_2}{K_1})^2)\alpha)}{(w + \frac{(1-w)K_2}{K_1})\frac{n+1}{n} - \frac{2}{n+1}(w + (1-w)(\frac{K_2}{K_1})^2)\alpha},$$

$$q_2^f = \alpha(n+1)\frac{(1-\alpha)nd\Delta^* - q_1^f \frac{K_2}{n+1}}{K_1},$$

where $K_1 = \alpha n + 1 - w(1-\alpha)$, $K_2 = n + 1 - w(1-\alpha)$.

Analysis of SPE

Proposition 4

Local equilibrium exists for any parameter combination. *Tables 1* and *2* show price reduction with respect to the standard Cournot auction. Stable local equilibrium corresponds to shaded blocks in the tables. Grey color corresponds to $q_2^f > 0$.

Table 1. $\frac{p^f - c}{\Delta^*}$ for $\alpha = 0.1$.

	w=0.1	w=0.2	w=0.4	w=0.6	w=0.8	w=0.9	$\frac{n+1}{n^2+1}$
n=2	1,04	0,95	0,78	0,66	0,57	0,53	0,60
n=3	1,06	0,95	0,76	0,63	0,54	0,5	0,40
n=4	1,08	0,94	0,74	0,6	0,51	0,47	0,29
n=5	1,10	0,94	0,71	0,57	0,47	0,43	0,23
n=6	1,12	0,92	0,67	0,52	0,43	0,39	0,19
n=7	1,13	0,9	0,62	0,47	0,37	0,34	0,16
n=8	1,13	0,86	0,55	0,40	0,31	0,28	0,14
n=9	1,11	0,77	0,45	0,31	0,23	0,21	0,12
n=10	1,00	0,6	0,29	0,19	0,14	0,12	0,11

Analysis of SPE

Table 2. $\frac{p^f - c}{\Delta^*}$ for $\alpha = 0.4$.

	w=0.1	w=0.2	w=0.4	w=0.6	w=0.8	w=0.9	$\frac{n+1}{n^2+1}$
n=2	1,02	0,77	0,48	0,34	0,26	0,23	0,60
n=3	1,02	0,76	0,46	0,32	0,25	0,22	0,40
n=4	1,03	0,75	0,44	0,31	0,23	0,21	0,29
n=5	1,03	0,73	0,42	0,29	0,22	0,20	0,23
n=6	1,03	0,72	0,40	0,27	0,20	0,18	0,19
n=7	1,03	0,69	0,38	0,25	0,19	0,17	0,16
n=8	1,03	0,67	0,35	0,23	0,17	0,15	0,14
n=9	1,02	0,64	0,32	0,21	0,16	0,14	0,12
n=10	1,00	0,60	0,29	0,19	0,14	0,12	0,11

Conclusion

With the growth of α amount of consumers preferring to act at the spot market increases. Thus, the area of stable SPE becomes more broad and the reduction of market power is more intensive.

Similar situation for parameter w: for big enough w the reduction of market power is more intensive, than in Bushnell's model. For smaller w the reduction is less intensive and the SPE is often instable.

Calculations show that only situations when arbitrageurs buy contracts at the forward market give stable SPE.

We plan to compare the results of the current research with statistics from existing organized electricity markets.

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